

**AVAILABLE INFORMATION ON ECONOMIC MODELS**

### Single market CES COMPAS model<sup>1</sup>

The CES COMPAS model assumes that the market being considered has a set of competitive sub-markets for each of a group of imperfect substitutes, usually differentiated by country of origin. Following Armington (1969), it assumes that buyers have well behaved, weakly separable preferences over products within an industry with similar, but not identical products (based usually on country of origin) and market preferences over the different groups are of the constant elasticity of substitution (CES) form. Because preferences are weakly separable, demand for goods from each of the n different countries (or group of countries) is a function of industry prices and total industry expenditure alone as follows:

$$(1) \quad q_i = \beta_i^\sigma \left( \frac{P_i}{P} \right)^{1-\sigma} \frac{Y}{P_i}$$

for  $i=1,2,\dots,n$  where  $q_i$  is the market demand for products in the  $i$ th country,  $\beta_i$  is a constant,  $P_i$  is the consumer price,  $Y$  is total spending on products in the industry. A price index may be defined as follows:

$$(2) \quad P = \left( \sum_{j=1}^n \beta_j^\sigma P_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

and industry expenditure depends upon the general price level in the industry (as represented by the price

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<sup>1</sup> Most of this section (with the exception of references to exogenous aggregate demand and supply shocks) is based on pp. 139-142 of Francois and Hall (1997).

index) and an aggregate price elasticity of demand (  $\eta$  ) as below:

$$(3) \quad Y = kP^{\eta+1} + g$$

where k is a constant and g is the exogenous percentage growth in market demand.

Supply of each product in the industry is represented in Constant Supply Elasticity form:

$$(4) \quad q_i = k_i \hat{P}_i^{\varepsilon_i} + g_i$$

for  $i=1,2,\dots,n$  countries where  $k_i$  is a constant,  $\hat{P}_i$  is the supply (pre-tariff) price,  $\varepsilon_i$  is the price elasticity

of supply to the domestic market, and  $g_i$  is the exogenous percentage growth in supply.

The effects of an imposing duty may be estimated as follows:

$$(4') \quad q_i = k_i \left( \frac{P_i}{1+t_i} \right)^{\varepsilon_i} + g_i$$

for  $i=1,2,\dots,n$ .

Calibrating the model (converting quantities to units that leave all initial prices equal to one and

exogenous growth rates to zero) leads to constant values as follows:  $\beta_i^\sigma = \frac{q_i}{k} \equiv s_i$  from equation (1);

$k=Y$  from equation (3); and  $k_i = q_i$  from equation (4). Combining (1) and (4'), substituting (3) into (1),

and rearranging, we have the following equilibrium condition:

$$(5) \quad \left( \frac{P_i}{1+t_i} \right)^{\varepsilon_i} + g_i = \left( \frac{P_i}{P} \right)^{1-\sigma} \frac{(P^{\eta+1} + g)}{P_i}$$

### Multi-market CES COMPAS model

Using the basic framework of the CES COMPAS model, the multi-market CES COMPAS model accounts for changes in the prices of upstream and downstream goods. The effect of changes in prices of downstream goods are accounted for through the aggregate demand function. An increase in the price of a downstream good increases aggregate demand for all of its inputs depending on the change in the general price level (as represented by its price index) of each downstream good (  $P_j^D$  ) and their respective cross price aggregate demand elasticities (  $\eta_j$  ) as below:

$$(3') \quad Y = kP^{\eta+1} \prod_{j=1}^m P_j^{D\eta_j} + g$$

where k is a constant and j=1,..., m downstream goods.

The effect of changes in prices of upstream goods are accounted for through the supply function. An increase in the price of an upstream good increases costs for downstream goods it is used to produce depending on the change in the general price level in the industry (as represented by its price index) for each upstream good (  $r_k$  ) and their respective cross price supply elasticity (  $\varepsilon_{ik}$  ) as below:

$$(4'') \quad q_i = k_i \left( \frac{P_i}{1+t_i} \right)^{\varepsilon_i} \prod_{k=1}^{m'} r_k^{\varepsilon_{ik}} + g_i$$

for  $i=1,2,\dots,n$  countries and  $j=1, 2,\dots,m'$  upstream goods.

Calibrating as above and then combining (1) and (4"), substituting (3') into (1), and rearranging, we have the following equilibrium condition:

$$(5') \quad \left( \frac{P_i}{1+t_i} \right)^{\varepsilon_i} \prod_{k=1}^{m'} r_k^{\varepsilon_{ik}} + g_i = \left( \frac{P_i}{P} \right)^{1-\sigma} \frac{P^{\eta+1} \prod_{j=1}^m P_j^{D_{\eta_j}} + g}{P_i}$$

### **An application to the U.S. flat rolled steel market**

The multi-market CES COMPAS model was used to simultaneously model both the direct effect of imposing trade remedies on six flat rolled steel products for which the Commission made either an affirmative or tied serious injury finding in the steel section 201 case and the indirect effects on flat rolled steel products which are either upstream or downstream from them.<sup>2</sup> The vertical linkages between these flat-rolled products are important to take into account because a large share of some flat rolled steel products are used to produce other flat-rolled products, and also make up a large share of their costs.

Figure 1 shows that about 90 percent of slab produced in the U.S. is used to produce hot-rolled steel (including coils), while the remaining 10 percent is used to produce cut to length plate. Also, about 55 percent of hot-rolled steel produced in the U.S. is used to produce cold-rolled steel. About 55 percent of cold-rolled steel is used to produce coated (galvanized) steel and about 10 percent of cold-rolled steel

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<sup>2</sup> The Commission made affirmative serious injury findings for five products: slabs, cut-to-length plate, hot-rolled steel, cold-rolled steel and coated steel. In addition the Commission's serious injury determination for tin products was tied.

is used to produce tin mill products. Figure 2 shows that the upstream products also make up a sizable share of their costs of product of the downstream goods.<sup>3</sup> Assuming that all reported material costs are for the upstream steel product, the cost share of upstream steel products range from 33 percent to 47 percent.

If the cost shares are the same for imported steel as they are for domestically produced flat-rolled steel products and they are unaffected by changes in prices or quantities for the goods in question, the domestic supply cross price elasticity between the downstream product and its corresponding upstream products should be the negative of the product of the domestic supply elasticity of the downstream good and the cost share  $(csh_k)$  of the upstream good,  $k=1, \dots, m$ .

$$(6) \quad \mathcal{E}_{1k} = - \mathcal{E}_1 * csh_k$$

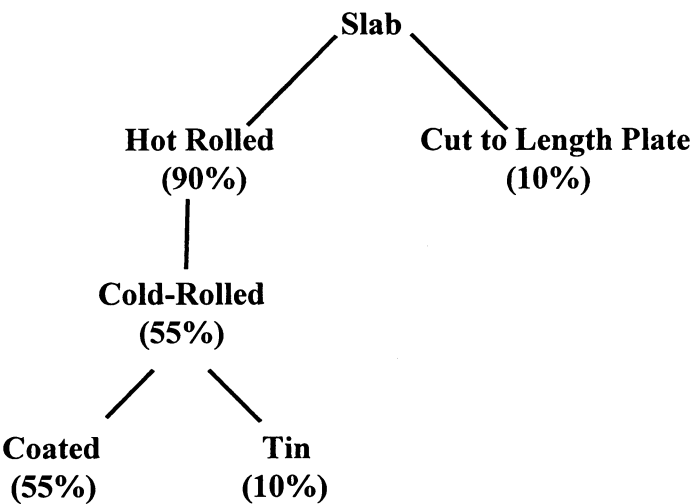
Similarly, if this is also true for the share of upstream goods used to produce downstream goods, the aggregate demand cross price elasticity between the upstream product and each of its corresponding downstream products should be equal the negative of the product of the aggregate demand elasticity of the downstream product and the usage share  $(ush_j)$  of each upstream product,  $j=1, \dots, m$ .

$$(7) \quad \eta_j = - \eta * ush_j$$

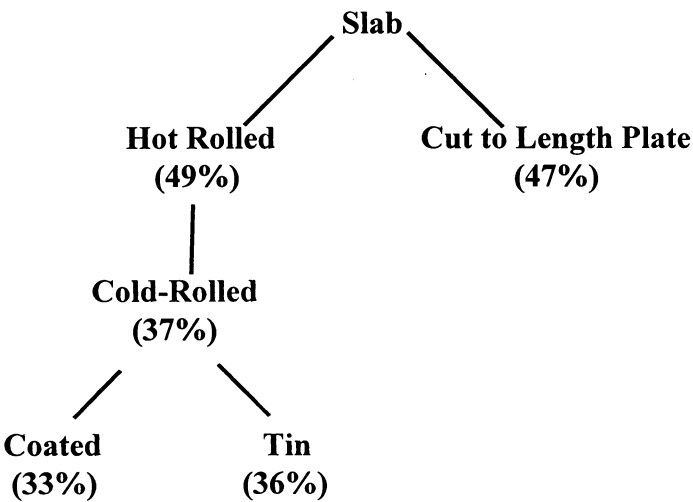
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<sup>3</sup> Upstream steel costs are assumed to be 90 percent of the material cost for coated steel and tin mill products.

**Figure 1- Share of Upstream Products Used in Downstream Products**



**Figure 2- Cost Share of Upstream Products in Downstream Products**



## References

Armington, P., "A theory of demand for products distinguished by place of origin." IMF Staff Papers, 16:159-178, 1969.

Francois, Joseph F. and Hall, H. Keith., "Partial Equilibrium Modeling," In Applied Methods for Trade Policy Analysis, edited by Francois, Joseph F. and Reinert, Kenneth P. Cambridge University Press, Cambridge, 1997.